Navigation Tracking with Multiple Baselines Part I: High-Level Theory and System Concepts

Kar-Ming Cheung **Charles Lee**
Jet Propulsion Laboratory, California Institute of Technology

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Outline of Talk

Motivations of Study

Deep Space "Real-Time" Positioning Using Simultaneous Baselines

- ΔDOR Theory for the General Case of N Simultaneous Baselines
- In-Depth Consideration of the Special Case of Two Baselines, and Insights

Near-Earth Application to Detect Uncooperative Spacecraft at GEO

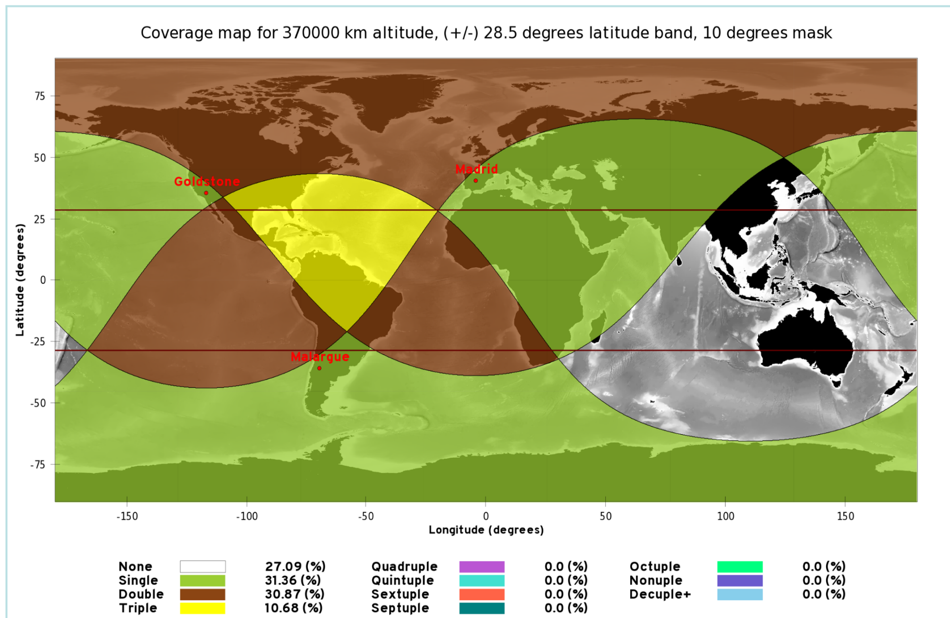
- Current Bistatic and Multistatic Radar Approach – Sum of Delays
- Proposed Relative Positioning Approach: Differential Radar – Difference of Delay

Preview of Results for Part II (Not in Paper)

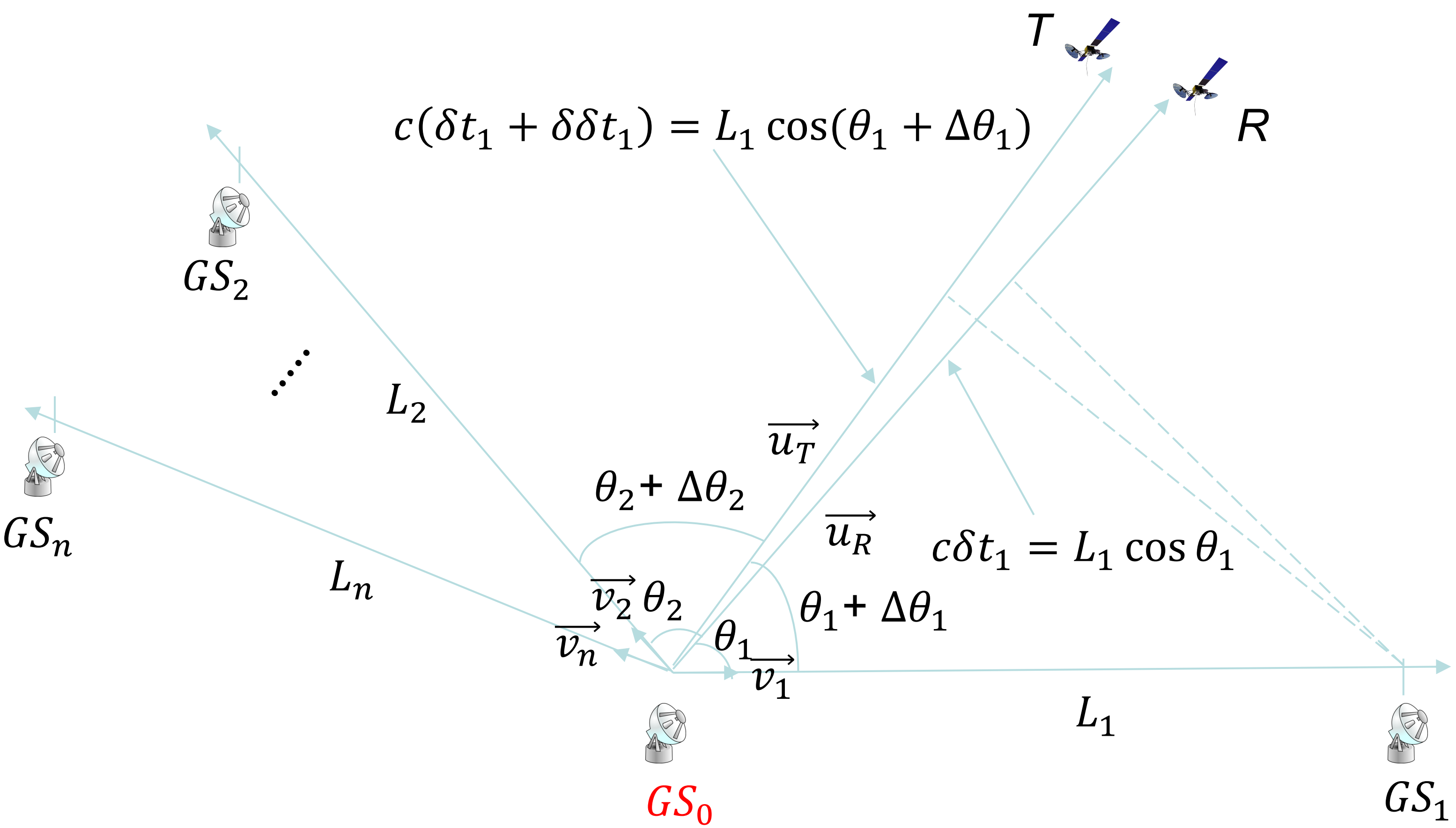
Conclusion and Future Work

Motivations of Study

- *Δ*DOR/SBI uses double-differencing to eliminate systematic biases in measurements
- The three DSN sites cover three approximately equally-spaced longitudes to provide near-continuous coverage of deep space
- Spacecraft occasionally see two DSN sites simultaneously, but never three
- Currently *Δ*DOR and SBI perform measurements on one baseline per pass
- Right ascension and declination are estimated using measurements from multiple passes, and the process is non-real-time
- Recent additions of non-DSN deep space antennas and increased cross-support enable spacecraft seeing two or more baselines simultaneously, e.g. Goldstone, Madrid, and Malargue



ΔDOR/SBI Theory for Multiple Baselines (1)



ΔDOR/SBI Theory for Multiple Baselines (2)

- Time Delay of Arrival (TDOA) from reference spacecraft R

$$c\delta t_i \approx L_i \cos\theta_i \quad \text{for } i = 1, 2, \ldots, n \qquad (1)$$

- TODA from target spacecraft T

$$c(\delta t_i + \delta\delta t_i) \approx L_i \cos(\theta_i + \Delta\theta_i) \quad \text{for } i = 1, 2, \ldots, n \qquad (2)$$

- Angle between target spacecraft and baseline i, denoted by $\theta_i + \Delta\theta_i$ for $i = 1,2, \ldots n$

$$\overrightarrow{u_T} \cdot \overrightarrow{v_i} = \cos(\theta_i + \Delta\theta_i) = \frac{c\delta\delta t_i + L_i \cos\theta_i}{L_i} + \epsilon_i \quad (3)$$

ϵ_i is a wavefront correction factor

- Constraint of unit vector, by definition

$$\overrightarrow{u_T} \cdot \overrightarrow{u_T} = 1 \qquad (4)$$

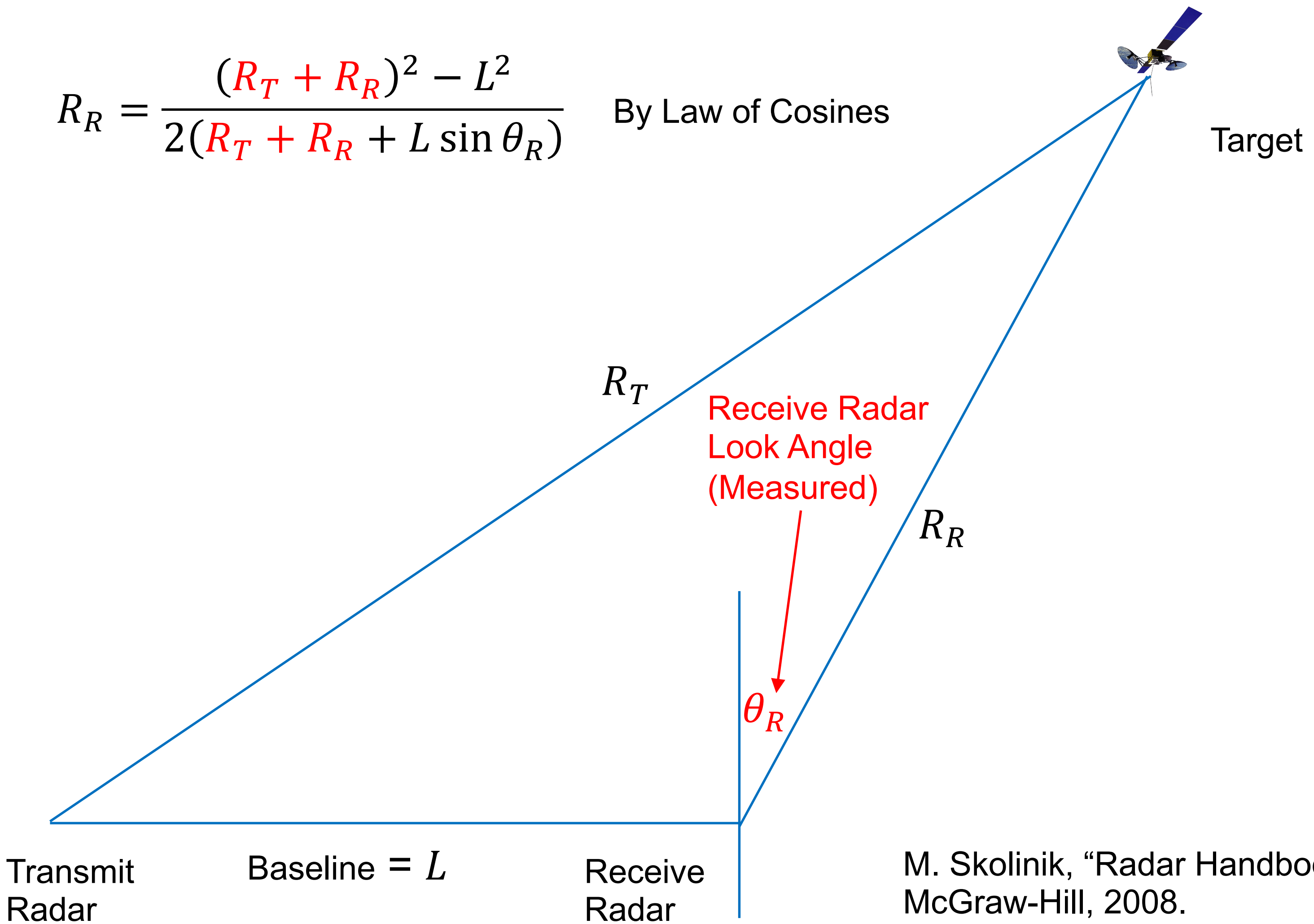
- $\overrightarrow{u_T}$ can be solved using Newton-Raphson Method on (3) and (3)
- For $n = 2$, and if a sufficient condition is met, explicit solutions exist

$$\overrightarrow{u_{T+}} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} Cos(\theta_1 + \Delta\theta_1) \\ \dfrac{\cos(\theta_2 + \Delta\theta_2) - \cos(\theta_1 + \Delta\theta_1)\cos(\psi)}{\sin(\psi)} \\ + \sqrt{1 - (\cos(\theta_1 + \Delta\theta_1))^2 - \dfrac{(\cos(\theta_2 + \Delta\theta_2) - \cos(\theta_1 + \Delta\theta_1)\cos(\psi))^2}{(\sin(\psi))^2}} \end{bmatrix} \quad \overrightarrow{u_{T-}} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} Cos(\theta_1 + \Delta\theta_1) \\ \dfrac{\cos(\theta_2 + \Delta\theta_2) - \cos(\theta_1 + \Delta\theta_1)\cos(\psi)}{\sin(\psi)} \\ - \sqrt{1 - (\cos(\theta_1 + \Delta\theta_1))^2 - \dfrac{(\cos(\theta_2 + \Delta\theta_2) - \cos(\theta_1 + \Delta\theta_1)\cos(\psi))^2}{(\sin(\psi))^2}} \end{bmatrix}$$

Near-Earth Current Multi-Static Radar Approach – Sum of Ranges

$$R_T + R_R = c\delta t_{rt} + L$$

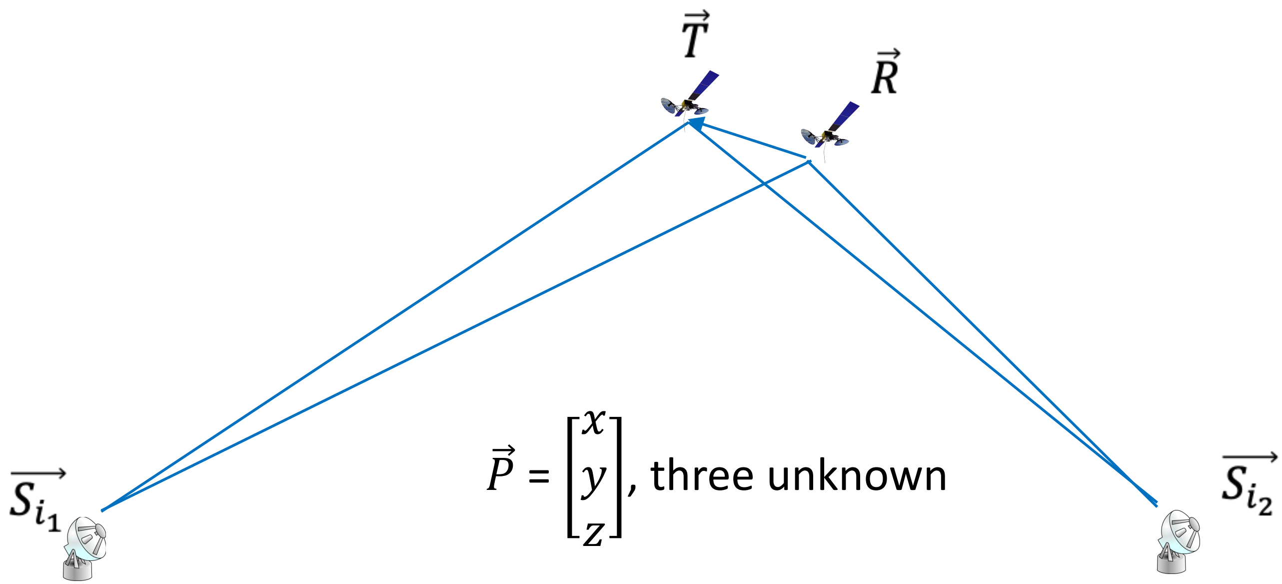
$$R_R = \frac{(R_T + R_R)^2 - L^2}{2(R_T + R_R + L\sin\theta_R)}$$ By Law of Cosines



M. Skolinik, “Radar Handbook”, 3rd Edition, McGraw-Hill, 2008.

New Near-Earth Multi-static Radar Approach – Difference of Ranges

- This approach requires a reference R in the vicinity of the target T
- In GEO above N. America, known GEO satellites are separated by 1/10 of a degree, thus providing many candidates as references to detect uncooperative spacecraft
- For the Moon, the Tycho Crater near the S. Pole is always facing Earth on the near-side, and has well-known radar signature for radio science calibration



$\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, three unknown

$$c\delta\delta t_i = \left(\|\overrightarrow{S_{i_1}R}\| - \|\overrightarrow{S_{i_2}R}\|\right) - \left(\|\overrightarrow{S_{i_1}T}\| - \|\overrightarrow{S_{i_2}T}\|\right)$$

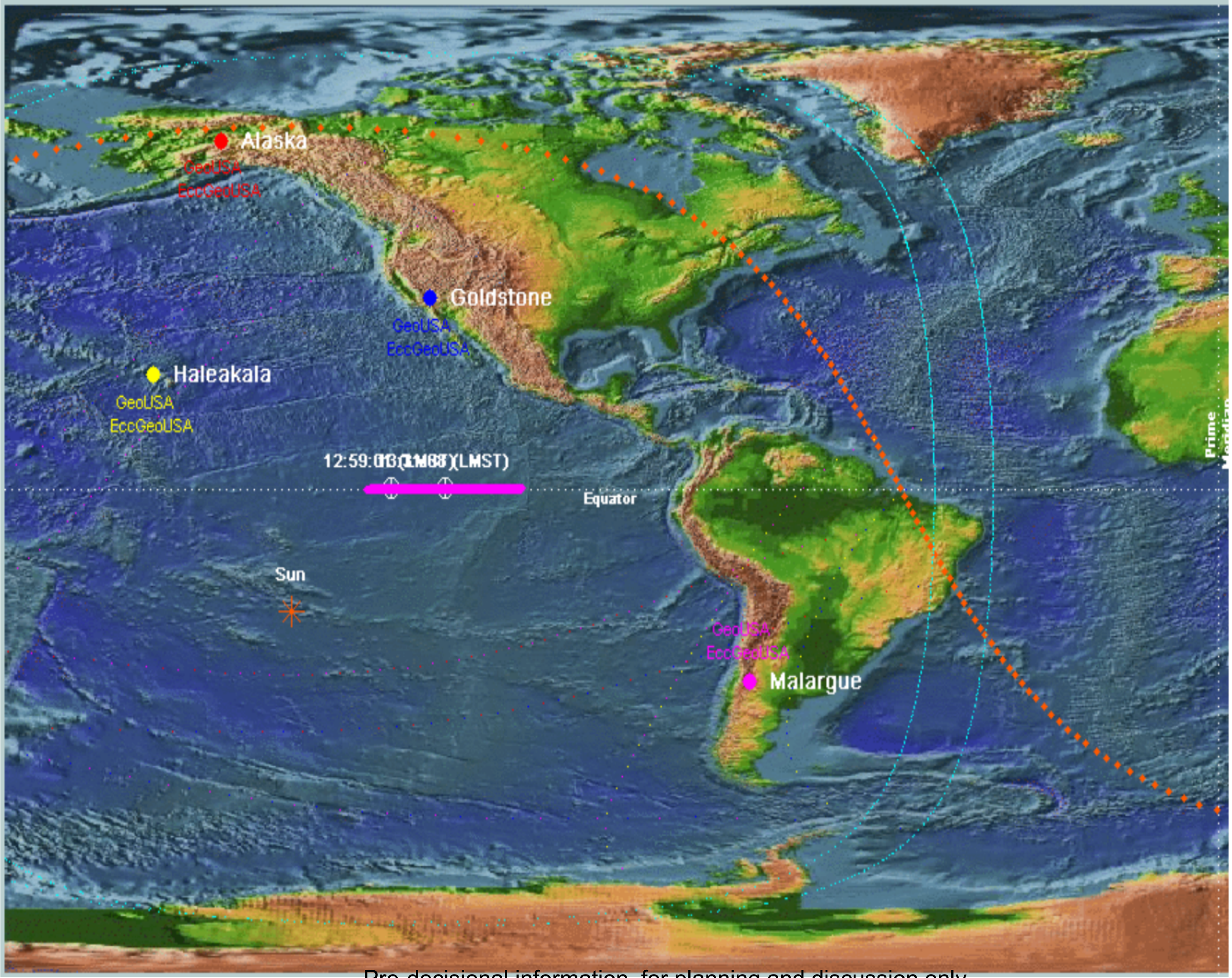
$$= \left(\|\vec{R} - \overrightarrow{S_{i_1}}\| - \|\vec{R} - \overrightarrow{S_{i_2}}\|\right) - \left(\|(\vec{R} - \overrightarrow{S_{i_1}}) + \vec{P}\| - \|(\vec{R} - \overrightarrow{S_{i_2}}) + \vec{P}\|\right)$$

for $i = 1, 2, \ldots, m$ $\quad m \geq 3$

$$\|\vec{P}\| = \sqrt{x^2 + y^2 + z^2}.$$

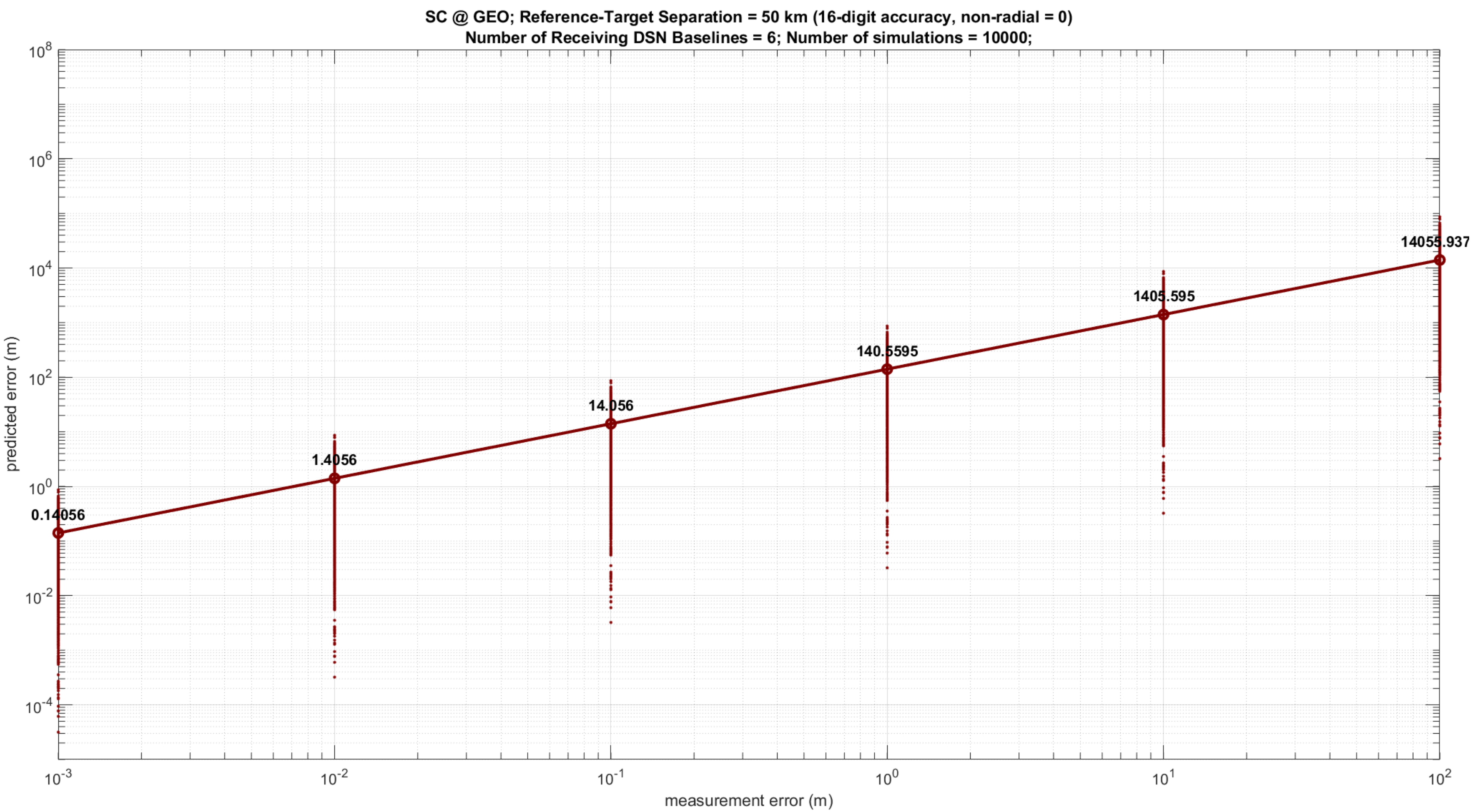
Preview of Performance for Part II (Not in Paper) (1)

- Consider the case of 4 ground stations at Goldstone, Alaska, Hawaii, and Malargue



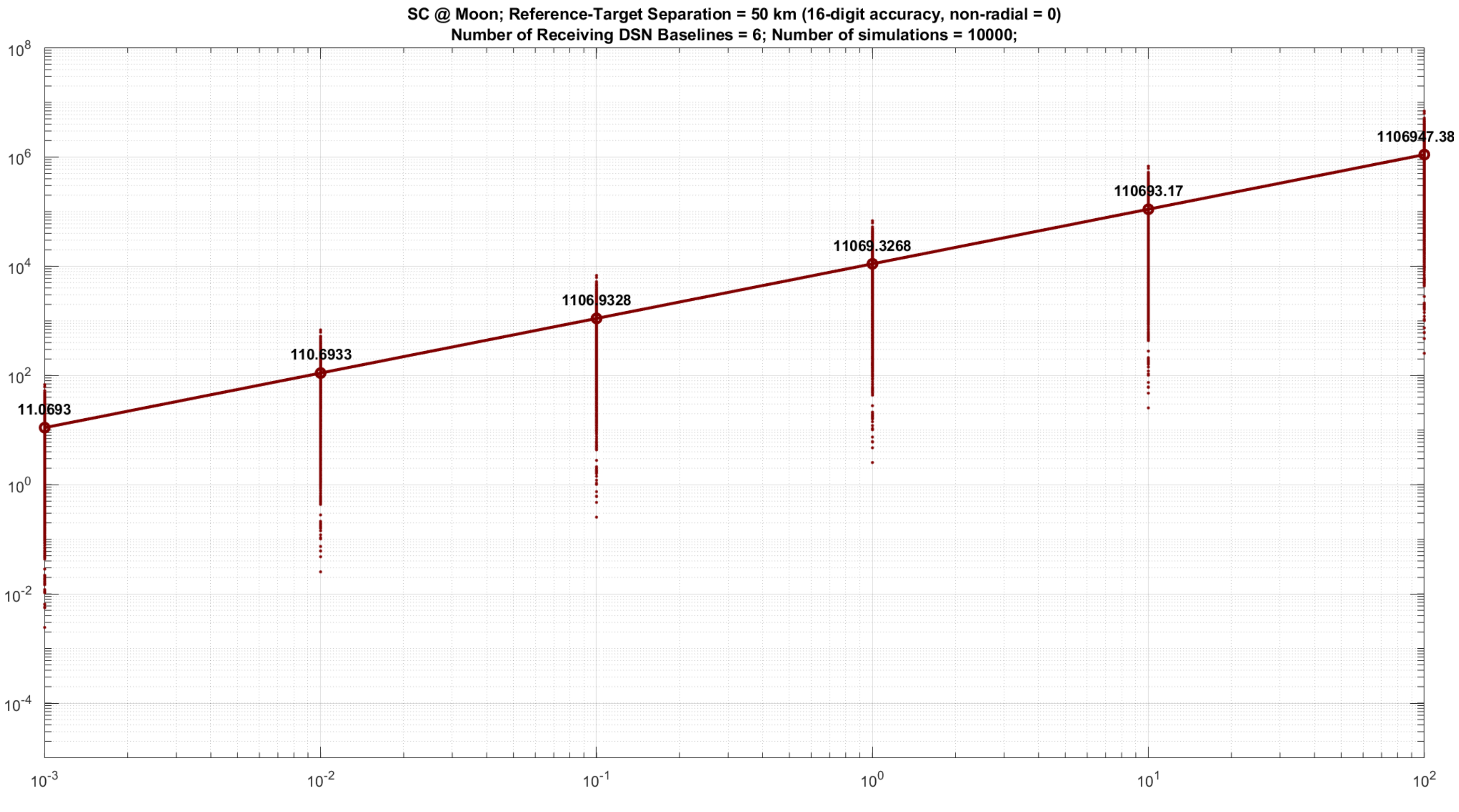
Preview of Performance for Part II (Not in Paper) (2)

- Double-differencing performance at GEO distance



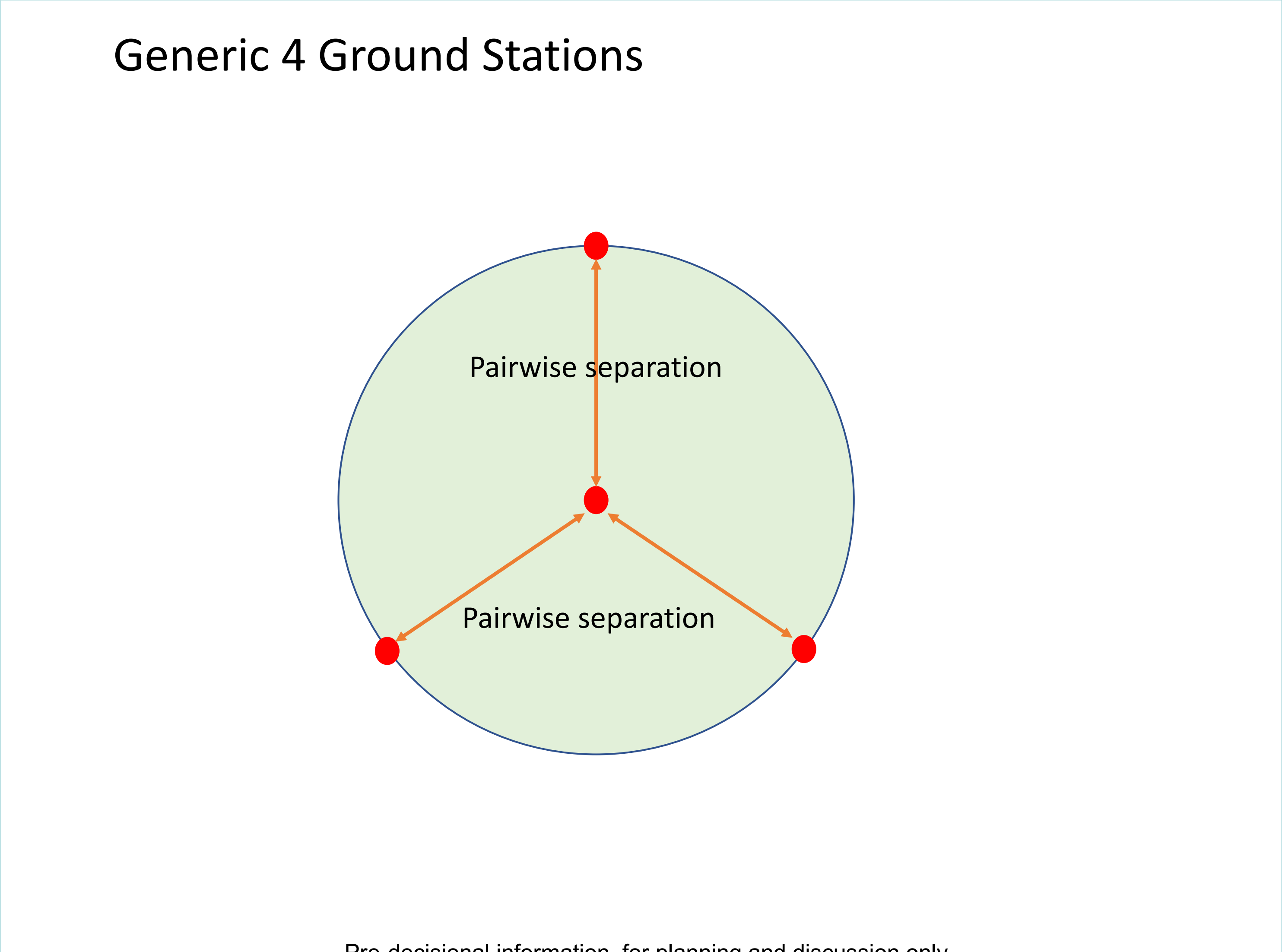
Preview of Performance for Part II (Not in Paper) (3)

- Double-differencing performance at Lunar distance



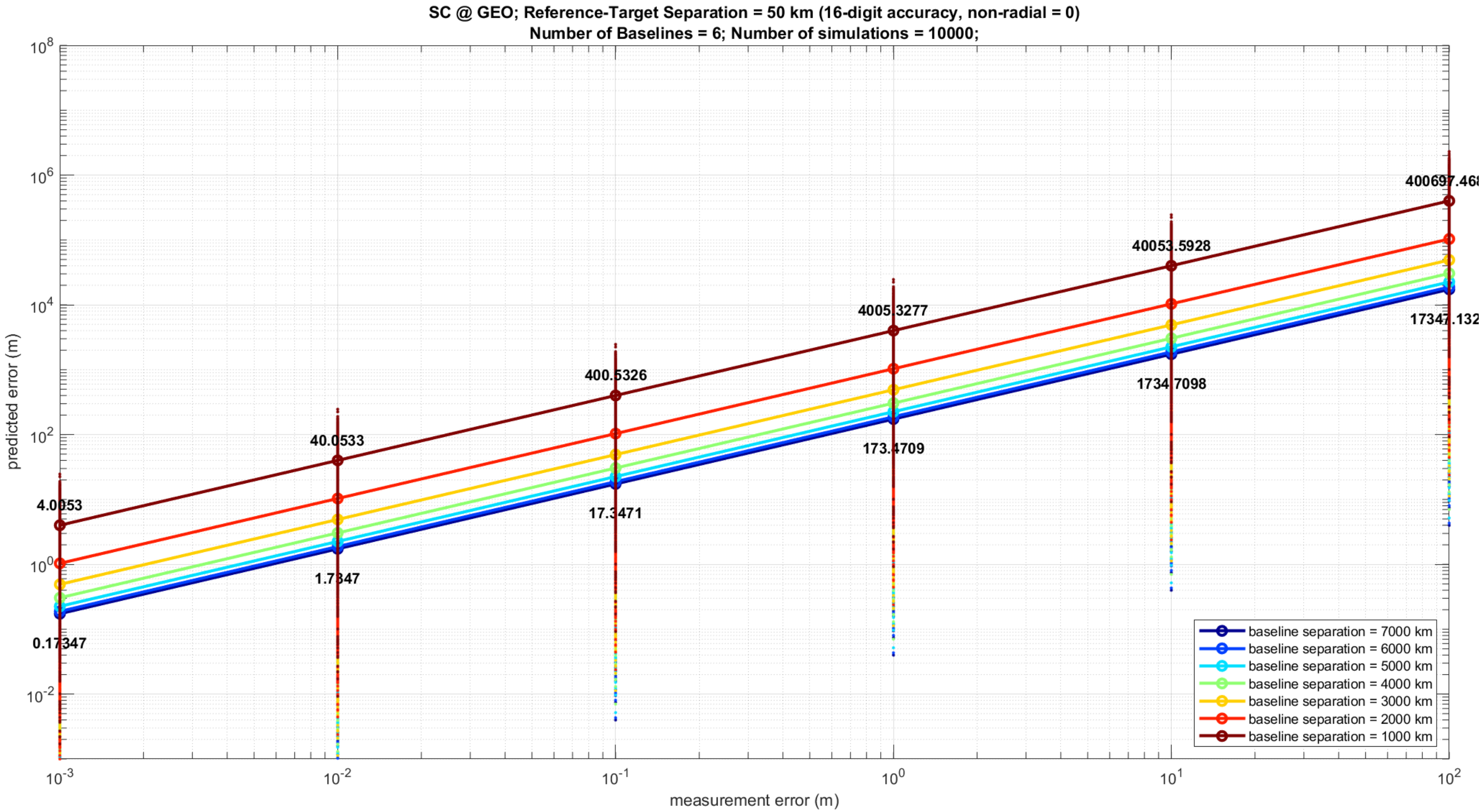
Preview of Performance for Part II (Not in Paper) (4)

- Consider the hypothetical case of 4 ground stations on a Sphere, and investigate the effect of station separation on accuracy performance



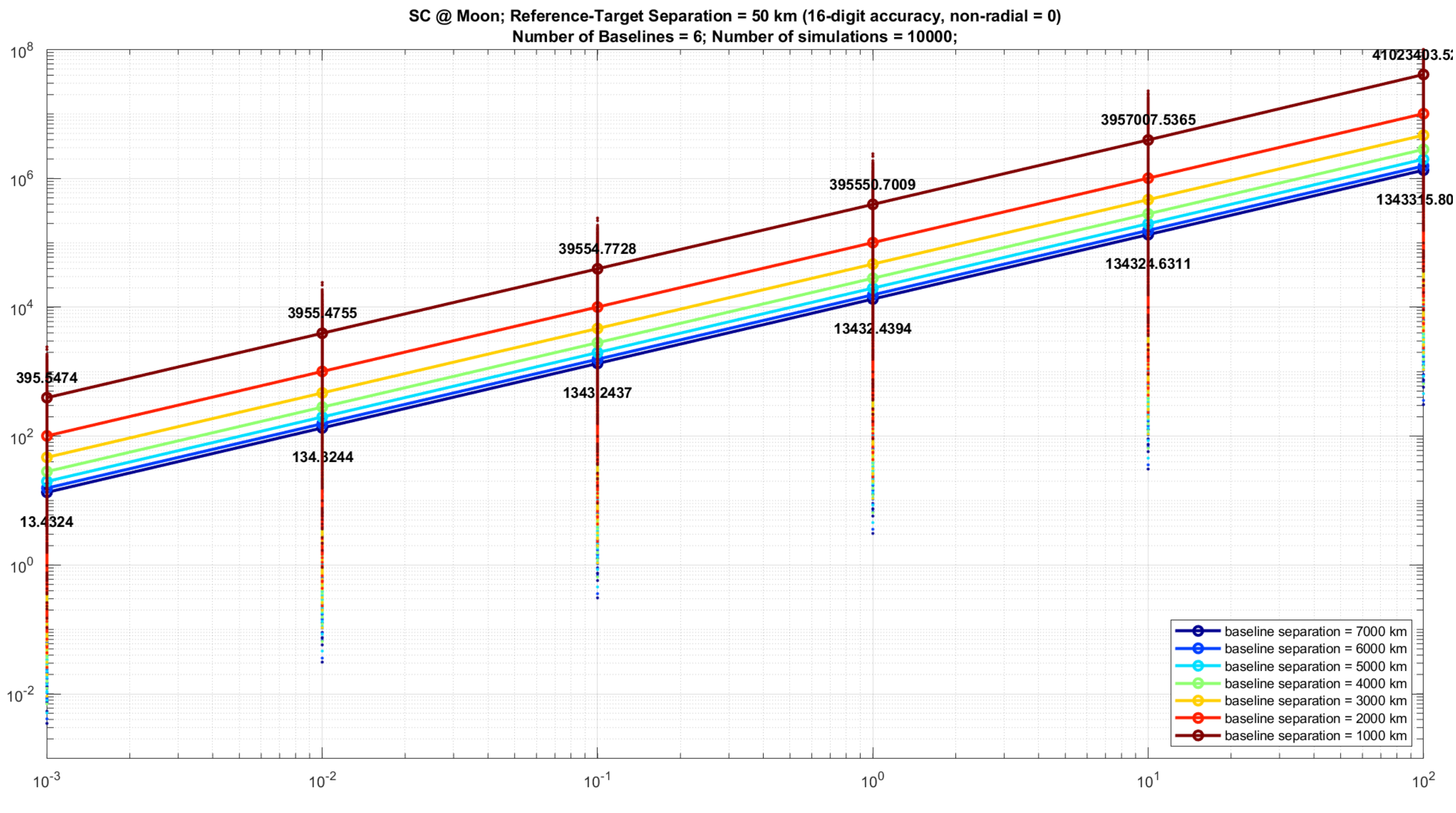
Preview of Performance for Part II (Not in Paper) (5)

- Double-differencing performance at GEO distance



Preview of Performance for Part II (Not in Paper) (6)

- Double-differencing performance at Lunar distance



Conclusion and Future Work

Conclusion

- This paper extends the current approach of ΔDOR and SBI that uses one baseline at a time (non-real-time), to simultaneously using multiple baselines (real-time)
- This paper introduces a near-Earth (or near-field) version of ΔDOR/SBI, and discusses a number of near-Earth applications

Path Forward

- Perform in-depth simulations to characterize the performances of deep space and near-Earth scenarios
- For near-Earth, introduce a "single-difference" approach that demonstrates sub-meter level accuracy at GEO and lunar distances
- Investigate the challenges of radar detection and target identifications at GEO and lunar distances
- Investigate the use of Kalman filters to improve the robustness and performance
- Investigate other near-Earth applications, e.g. fighter jet dog-fight, submarine cat-and-mouse, missile defense, multi-GNSS weak GPS at lunar distance, etc.
- Note: patent application pending